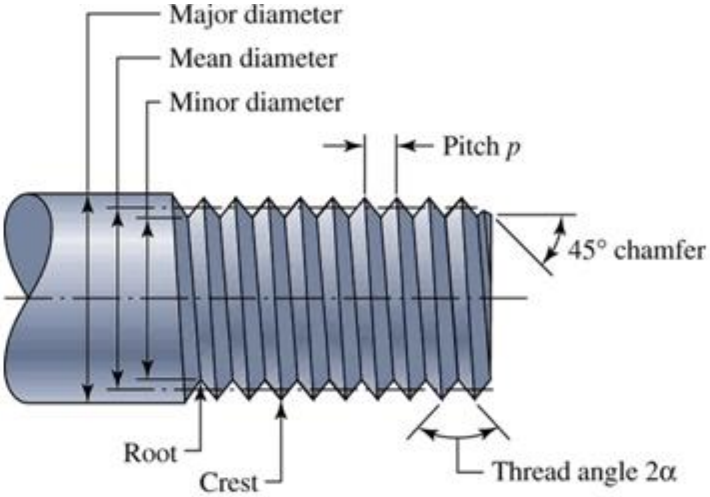
CH 8: Screws, Fasteners, and the Design of Non-Permanent Joints

This chapter introduces non-permanent joining elements such as bolts, nuts, setscrews rivets, pins, keys, etc.

It also introduced power screws which changes angular motion to linear motion, where it is similar in principle to screws and bolts.



Thread Standards and Definitions

- The terminology of screw threads is illustrated in the figure.

  Pitch (p): the distance between adjacent threads measured parallel to thread axis.

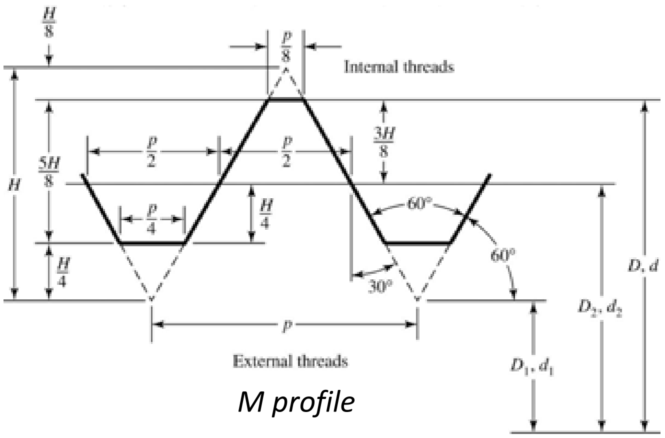
  Major diameter (D): the largest diameter of the screw thread.

  Minor diameter (D_1): also called "*root diameter*", is the smallest diameter of the screw thread.

  Mean diameter (D_2): also called "*pitch diameter*", the average diameter of the screw thread (*considering the theoretical full height of the threads*).

  Lead (l): the distance a nut moves parallel to the screw axis when it rotates one full turn.

- For a single thread screw the lead is same as the pitch.
- For multiple thread screws (*two or more threads run beside each other*) the lead equals the pitch multiplied by the number of threads.
- All threads are usually right-handed unless otherwise is indicated.
- Tensile tests showed that a threaded road has a tensile strength equal to that of an unthreaded rod having diameter equal to the average of the pitch diameter and minor diameter of the threaded rod.
- Bolts are standardized and there are two standards: *Metric* (***ISO***) and *American* (***Unified***). In both standards the thread angle is $60°$.



M profile

*Metric (**ISO**)*:

- There are two standard profiles ***M*** and ***MJ*** where both have a similar geometry but the ***MJ*** has a rounded fillet at the root and a larger minor diameter and therefore it has a better fatigue strength.
- Metric bolts are specified by the *major diameter* and the *pitch* (*both in mm*).
  - *Example*: $\underset{\nwarrow \text{Profile}}{M}10 \times 1.5$ (*10 mm* major diameter and *1.5 mm* pitch).

❖ *Table 8-1* gives the standard sizes of *Metric* bolts along with the effective tensile stress area and the root diameter area (*which is used when the bolt is subjected to shear loading*).

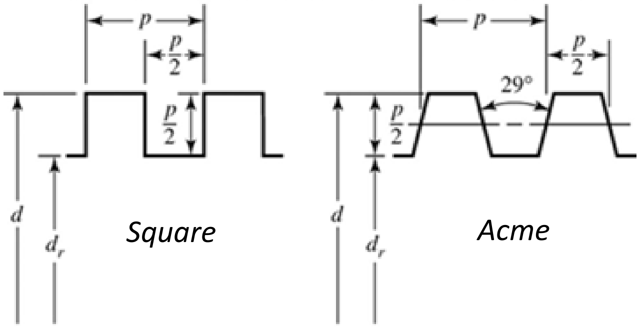
- Note that there is Coarse-pitch and Fine-pitch (*more threads*) where the fine-pitch has better tensile strength.

*American (**Unified**)*:

- There are two standard profiles ***UN*** and ***UNR*** where the ***UNR*** has a filleted root and thus better fatigue strength.
- Unified threads are specified by the *major diameter* (*in inch*) and the *number of threads per inch* (N).
  - *Example*: $\underset{\underset{\text{Diameter}}{\uparrow}}{¼} - \underset{\underset{(N)}{\uparrow}}{20}\ \underset{\underset{\text{Profile}}{\uparrow}}{U}N\underset{}{C} \xleftarrow{} \text{Coarse or } \mathbf{F} \text{ (Fine)}$

❖ *Table 8 -2* gives the standard sizes along with the tensile stress areas and root diameter areas (*used for shear loading*) for *Unified* bolts (*Coarse and Fine series*).

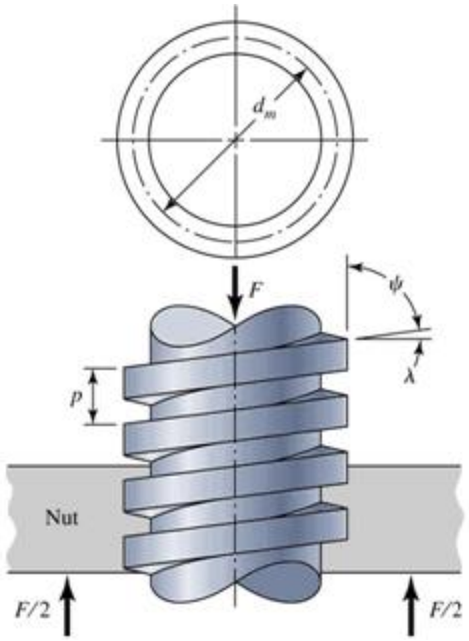
- Note that for diameters smaller than *1/4 inch*, the size is designated by size numbers rather than diameter.



- For screws used to transmit power (*Power Screws*) there are *Square* or *Acme* threads.

❖ *Table 8-3* gives the standard diameters and associated pitch for *Acme* thread power screws.

The Mechanics of Power Screws



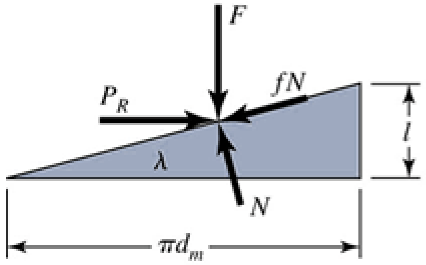
Power screws are used to change angular motion to linear motion. It is used in jacks, lathes, vises, etc.

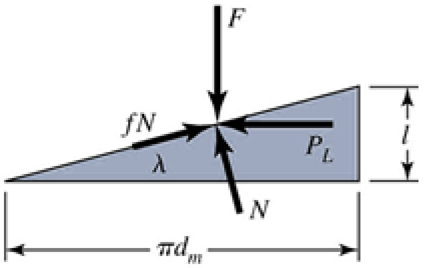
p (Pitch) = l (Lead: for single thread screws)

λ : Lead angle, ψ : Helix angle

d_m : Mean diameter

- To find the torque needed to raise the load (T_R) or needed to lower the load (T_L), let one thread of the screw to be unrolled (*assuming square thread*).





  - Using static equilibrium equations and knowing that $T = P(d_m/2)$ and $tan\,\lambda = l/\pi d_m$, we can find that:

    The torque needed to raise the load F:

$$\boxed{T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right)}$$

The torque is used to raise the load and to overcome thread friction

    The torque needed to lower the load F:

$$\boxed{T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right)}$$

The torque is used to overcome a part of the friction

- If T_L turns to be zero or negative this means that the screw will spin (*the load will be lowered*) without any external effort, and this is usually not desired.
  - ✓ In order to ensure that this will not happen, then we should have:

$$f > tan\,\lambda$$

Self-Locking condition

- The efficiency is important in evaluating power screws.
  - If $f = 0$ (*no friction*) then all the applied torque is transferred into force (*100% efficiency*) and the torque needed to rise the load T_R to becomes:

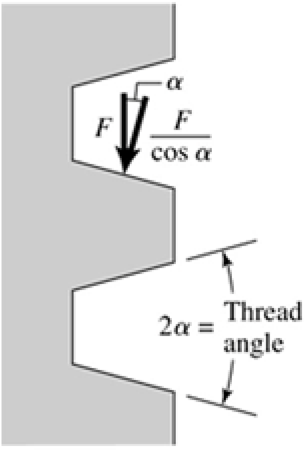
    $$T_o = \frac{Fl}{2\pi}$$

    Thus the efficiency is found as:

    $$e = \frac{T_o}{T_R} = \boxed{\frac{Fl}{2\pi T_R}}$$

- For screws with *Acme* thread, there is additional wedging force due to the angle α which increases the frictional forces (F becomes $F/\cos\alpha$).
  - Thus all frictional terms are divided by ($\cos\alpha$) therefore T_R becomes:

    $$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - fl \sec\alpha}\right)$$



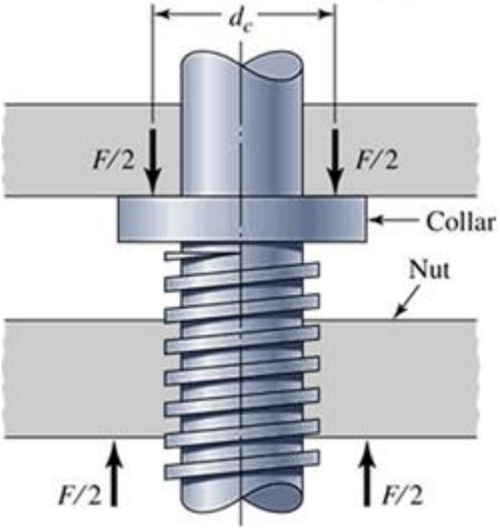
  - Due to the increased friction the efficiency of *Acme* thread is less than that of *Square* threads.
  - However *Acme* threads are commonly used because they are easier to machine and split-nuts (*to compensate for wear*) can be used.

- In many cases a Collar (*sliding friction bearing*) is used to support the load (as seen in the figure), and thus additional component of torque (T_c) is needed to overcome the friction between the collar and load plate.

  The collar torque is found as:

  $$\boxed{T_c = \frac{F f_c d_c}{2}}$$

  Where, f_c : coefficient of friction for the collar
  d_c: collar mean diameter

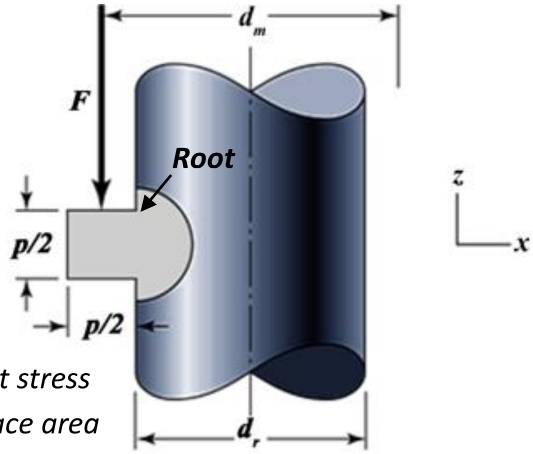


- ❖ *Table 8-5* gives the coefficients of sliding (*dynamic*) and starting (*static*) friction for some common metal pairs (*The best is for bronze on bronze, but since bronze have relatively low strength it is not commonly used for the screw*).
- ❖ *Table 8-6* gives the coefficients of friction (*sliding and starting*) for thrust collars.

- It is necessary to find the stresses developed in the power screw while performing its function to ensure its safety.
  - ▪ The stresses in the body of the power screw are found as:

Normal stress: $\sigma = \frac{F}{A} = \frac{4F}{\pi\ {d_r}^2}$ *Tension or Compression*

Shear due to the torque: $\tau = \frac{Tc}{J} = \frac{16T}{\pi\, {d_r}^3}$ *Maximum at the root*



  - ➢ If the screw is loaded in compression, then buckling should be considered also.
    - ✓ *Johnson* or *Euler* formula can be used according to the slenderness ratio (*use the root diameter*).
  - ▪ The threads are also subjected to stresses which are:

Bearing stress: $\sigma_B = \frac{-F}{A} = -\frac{2F}{\pi d_m n_t p}$ *Compressive contact stress over the entire surface area*

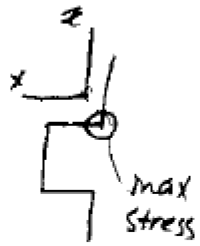
where n_t is the *number of engaged threads*

Bending stress: $\sigma_b = \frac{Mc}{I} = \frac{6F}{\pi d_r n_t p}$ *Max at the top surface of the root*

Transverse shear: $\tau = \frac{3V}{2A} = \frac{3F}{\pi d_r n_t p}$ *Max at the center of the root* & zero at the top surface

- Experimental results show that the load is not shared equally between the engaged threads, instead the *first takes 0.38 of the load*, 2nd takes *0.25*, 3rd takes *0.18*, and the 7th is free of load (*assuming the number of engaged threads is six or more*).
  - ➢ Thus, the highest stresses are at the root of the first thread, and in the analysis we do not divide the load by the number of engaged threads (n_t) but rather we do the analysis based on *38%* of the load.

- The critical stress occurs at the top of the root and it's found according to *Von Misses* knowing that:



$$\text{Bending} \longrightarrow \quad \sigma_x = \frac{6F}{\pi d_r n_t p} \quad , \quad \tau_{xy} = 0$$

$$\sigma_y = 0 \quad , \quad \tau_{yz} = \frac{16T}{\pi\, {d_r}^3} \longleftarrow \text{Torsion}$$

$$\text{Axial} \longrightarrow \quad \sigma_z = \underset{\uparrow}{-}\frac{4F}{\pi\, {d_r}^2} \quad , \quad \tau_{zx} = 0$$

Critical when this is compression and bending is (+)

See ***Example 8-1*** *from text*

Threaded Fasteners

The purpose of a bolt or screw is to clamp "*fasten*" two or more parts together.

- The dimensions of bolts and screws are standardized and there are several head styles that are being used (*Figures 8-9, 8-10 and 8-11 show some of the common head styles for bolts and cap screws*).
  - The terms bolt and screw are sometimes used interchangeably and they can refer to the same element. In general, a bolt is used with a nut while a screw is used with a threaded hole (*Not a standard definition*).
- Washers must be used under bolts heads in order to prevent the sharp corner of the hole from biting into bolt head fillet where that increases stress concentration.
  - *Tables A-29* and *A-30* give the standard dimensions for bolt heads.
  - *Table A-31* gives the standard dimensions of hexagonal nuts.
  - *Tables A-32* and *A-33* give the standard dimensions of plain washers.

- The length of a bolt is not chosen arbitrarily, usually the length is chosen from the preferred sizes given in *Table A-17*.
- The *length of the threaded portion of a bolt* (L_T) is also standardized where the relation for metric sizes is given as:

$$L_T = \begin{cases} 2\,d + 6\,mm & L \le 125\,, d \le 48\,mm \\ 2\,d + 12\,mm & 125 < L \le 200\,mm \\ 2\,d + 25\,mm & L > 200\,mm \end{cases}$$

For sizes in inch see Table 8-7

Bolts Strength

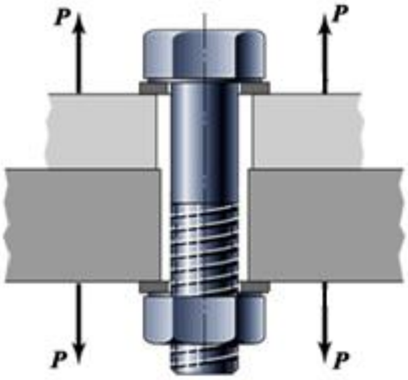
According to standards, tensile stress in the bolt (*due to both preload and external load*) should not exceed the minimum *Proof Strength* "S_p" of the bolt where the *Proof Strength* is defined as the maximum stress value that the bolt can withstand without having a permanent deformation (*it is slightly less than the Yield Strength where it corresponds to the Proportional Limit*).

- Bolt materials are standardized and they are classified into different grades.
  - ❖ *Table 8-9* gives the *SAE* specifications for steel bolts.
  - ❖ *Table 8-10* gives the *ASTM* specifications for steel bolts.
  - ❖ *Table 8-11* gives the *ISO* specifications for metric steel bolts.

Note the head markings of the bolts

Joints with External Load (*Tension Joints*)

Bolts and screws are used to clamp two, or more, parts together where these parts are subjected to an external force trying to separate them.



- ▪ When a bolt and a nut are used to make a joint, the nut is usually tightened to grip the joint firmly.
- ▪ This tightening of the nut introduces a tensile force in the bolt (*called the pre-load*) and a compressive force "of the same value" in the clamped material.
- ▪ When the external "*separating*" force is applied to the joint it will be divided between the bolt (*where it increases the tension in the bolt*) and the clamped material (*where it reduces the compression in the material*).
- ▪ In order to find the portion of the external load carried by the bolt and the portion carried by the material, spring methodology is used where the bolt and the clamped material are represented as two springs in parallel.
- ▪ Therefore, the portion of the external load carried by each of the two springs (*representing the bolt and the clamped material*) depends on the stiffness (*spring rate*) of each of the two springs.

Joints – Fastener Stiffness

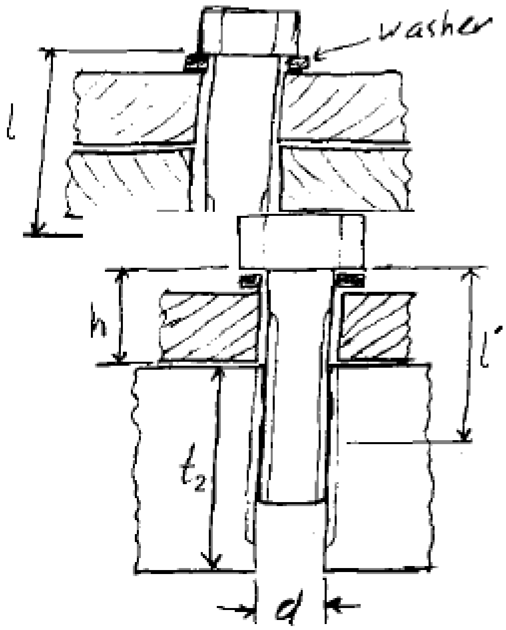
The stiffness (*spring rate*) is the ratio of applied force to the deflection caused by that force.

- The spring rate of axially loaded members is defined as:

$$k = AE/l \qquad \text{(from Chapter 4)}$$

  - For bolts and screws the length of the member that is being subjected to the tensile load is called the "*grip*" (l).
    - For a bolt-nut connection, the grip (l) is the total thickness of the clamped material (*including the washers*).
    - For cap screw connection, the "*effective grip*" (l') is found as:

$$l' = \begin{cases} h + t_2/2 & t_2 < d \\ h + d/2 & t_2 \geq d \end{cases}$$



  - In general, the bolt will have threaded and unthreaded portions where each will have a different stiffness (*because the cross-sectional area is different*). Thus, the two portions are treated as two springs in series, and the total stiffness is found as:

$$1/k = 1/k_1 + 1/k_2$$

Knowing that,

$$k_t = \frac{A_t E}{l_t} \qquad \& \qquad k_d = \frac{A_d E}{l_d}$$

Stiffness of the threaded portion *Stiffness of the unthreaded portion*

  - Thus the *effective stiffness* of the bolt or cap screw is found to be:

$$\boxed{k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}}$$

Where l_t: length of threaded portion of the grip.

l_d: length of unthreaded portion.

A_d: "major-diameter area" of fastener.

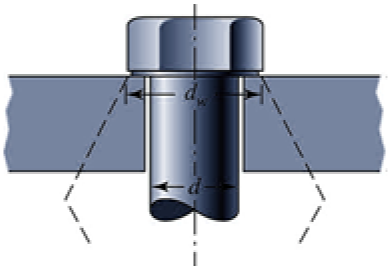
A_t: tensile stress area (*Tables 8-1 & 8-2*).

Joints – Member Stiffness

The clamped members will be treated as springs in series in order to find the total stiffness.

$$1/k = 1/k_1 + 1/k_2 + 1/k_3$$

- If one of the clamped members is a soft gasket (*which has a very small stiffness compared to other members*), the stiffness of other members can be neglected and only the gasket stiffness is used.



- Experimental investigation showed that the area subjected to compressive stress in the clamped zone has a conical shape with half apex angle of about *30*˚.
    - For members made of the same material (*same* E), the effective stiffness is found to be:

$$k_m = \frac{\pi\, Ed \tan\gamma}{2\, ln \dfrac{(l \tan\gamma + d_w - d)(d_w + d)}{(l \tan\gamma + d_w + d)(d_w - d)}}$$

-
    - Knowing that the standard washer diameter is *50%* greater than the bolt diameter ($d_w = 1.5d$) and $\gamma = 30°$, the effective member stiffness can be simplified as:

$$\boxed{k_m = \frac{0.5774\, \pi\, E\, d}{2\, ln\left(5\, \dfrac{0.5774\, l + 0.5\, d}{0.5774\, l + 2.5\, d}\right)}}$$

Where (l) is the total thickness of the clamped members.

- Alternatively, k_m can be found using a curve fit equation (*obtained from FEA*) which gives a very close value:

$$\boxed{k_m = Ed\left[A\, e^{Bd/l}\right]}$$

- The <u>constants A and B</u> depend on the material being used and their values are given in *<u>Table 8-8</u>* (*both are A and B unitless*).

<u>Tension Joints – The External Load</u>

When an <u>external tensile load</u> "P" is applied to the joint, the load will be <u>divided</u> between the bolt and the clamped member (*as long as the load "P" is not large enough to separate the clamped members*), <u>in addition to the preload</u> "F_i" carried by each.

Defining;
P_b : portion of P taken by bolt.
P_m : portion of P taken by members.
$F_b = P_b + F_i$: resultant bolt load.
$F_m = P_m - F_i$: resultant members load.
$C = P_b/P$: fraction of external load carried by bolt.
$(1 - C)$: fraction of external load carried by member.

- Since the bolt and members will have the same deflection:

$$\delta = \frac{P_b}{k_b} = \frac{P_m}{k_m} \quad \Rightarrow \quad P_b = P_m \frac{k_b}{k_m}$$

Knowing that,

$$C = \frac{P_b}{P} = \frac{P_b}{P_b + P_m} = \frac{P_m \frac{k_b}{k_m}}{P_m \frac{k_b}{k_m} + P_m}$$

$$\Rightarrow \quad \boxed{C = \frac{k_b}{k_b + k_m}}$$

The <u>Stiffness Constant</u> of the joint

And the resultant <u>bolt load</u> is:

$$F_b = P_b + F_i = \boxed{CP + F_i}$$

And the resultant <u>member load</u> is:

$$F_m = P_m - F_i = \boxed{(1 - C)P - F_i}$$

- Note that these relations are valid only when ($F_m < 0$), meaning that the members are still under compressive load and did not get separated.

- If the external load is large enough to separate the members, then the entire load will be carried by the bolt: $\boxed{F_b = P}$ (*this should not happen*).

Statically Loaded Tension-Joints with Preload

- For a bolt subjected to an external load "P" and having a preload "F_i", the tensile stress is found as:

$$\sigma_b = \frac{C\,P + F_i}{A_t}$$

  - The limiting value of σ_b is the *Proof Strength* "S_p", therefore the static factor of safety *"n_p"* can be calculated as:

$$n_p = \frac{S_p}{\sigma_b} = \frac{S_p\,A_t}{C\,P + F_i}$$

  - However, such factor of safety is not very useful since it is expressed based on the stress in the bolt which is due to both the pre-load and external load. A more useful definition will be a "*load factor*" of safety that indicates how many times the external load can be increased (since the pre-load value will remain constant) and that can be found as:

$$\frac{C\,(n_L P) + F_i}{A_t} = S_p$$

Thus,

$$\boxed{n_L = \frac{S_p A_t - F_i}{C\,P}}$$

Where n_L is the Load Factor (*i.e. the load "P" can be increased "n_L" times for the stress to reach S_p*).

- It is also necessary to ensure that separation will not occur (*if separation occurs, the bolt will carry the entire load*).
  - Separation occurs when:

$$F_m = 0 = (1 - C)n_o P - F_i$$

Thus,

$$\boxed{n_o = \frac{F_i}{P(1-C)}}$$

Where n_o is the *Load Factor guarding against joint separation*.

- ➢ Both load factors n_L & n_o should be calculated, and the smaller of the two will be the load factor of safety for the joint.

- According to standard, the recommended value of preload is given as:

$$\boxed{F_i} = \begin{cases} 0.75\, F_p & for\ nonpermanent\ connections\ (reused\ fasteners) \\ 0.9\, F_p & for\ permanent\ connetions \end{cases}$$

Where F_p is the *Proof Load*: $\boxed{F_p = S_p A_t}$

- ➢ If the preload "F_i" is set according to the recommended value, then it is less likely that separation will happen before the stress reaches the proof strength (*unless high strength fasteners are used*).

Relating Bolt Torque to Bolt Tension (*Preload*)

Applying preload to the bolt (*by tightening*) is very important where it increase the strength of the joint by preventing separation of the members.

- It is important to relate the torque used in tightening the bolt to the amount of the preload developed in the bolt in order to ensure that the preload is sufficient and that it did not exceed the allowable value.
- The relation between torque and preload is given as:

$$\boxed{T = K\, F_i d}$$

Where "K" is the *Torque Coefficient* and it is given as:

$$K = \left(\frac{d_m}{2d}\right)\left(\frac{\tan\lambda + f\sec\alpha}{l - f\tan\lambda\sec\alpha}\right) + 0.625 f_c$$

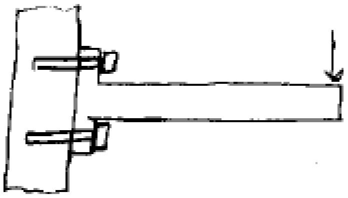
- ▪ Note that this is similar to the relation used for power screws which *Acme* threads. The angle α is the thread angle where its standard value is *30*° and f_c is the coefficient of friction between the bolt head or nut and the clamped material or washer.

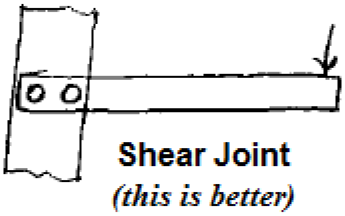
  - On average, $f = f_c = 0.15$ and this gives a torque coefficient of $K = 0.2$ regardless of the bolt size.
- ❖ *Table 8-15* gives the K values for different types of bolts.

See ***Example 8-4*** *from text*

Shear Joints

In many cases, joints can (*and should*) be loaded in shear only such that no additional tensile stress is introduced in the fasteners (*only the preload*).



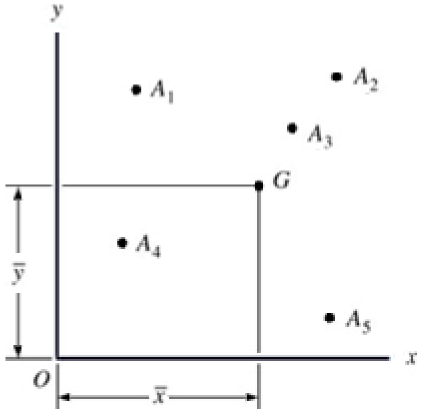


- In shear joints, the shear load is carried by the friction between the members which is introduced by the clamping force (*the preload*).
- If the friction is not sufficient, the shear will be carried by the fasteners (*in reality even when a number of fasteners are used at the joint, only two fasteners will carry the load because of the errors in holes size and location*).
- If locational-pins (*dowel pins*) or rivets are used for the shear joint, the shear load will be distributed between them.

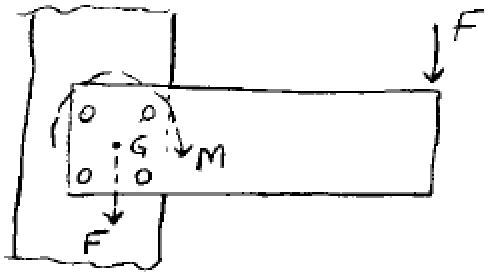
- In order to analyze the shear joints subjected to moment, the *relative center of rotation* between the two members needs to be determined.
  - For a pattern of fasteners of different sizes, the center of rotation (*Centroid*) is found as:

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$



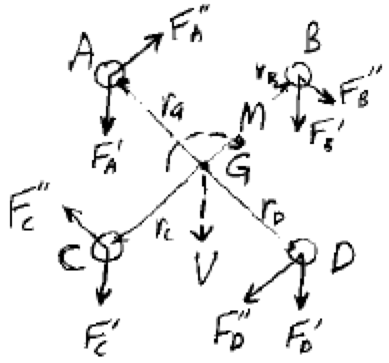
- For a shear joint loaded by a shear force and a moment, each fastener will carry two shear components:
  - Primary shear (*due to the shear load*).
  - Secondary shear (*due to the moment*).



- The shear load will be divided evenly between the fasteners (*assuming all fatteners have the same area and same* E) and each will have:

Primary shear $$\boxed{F_n' = {}^{V}/_{n}}$$

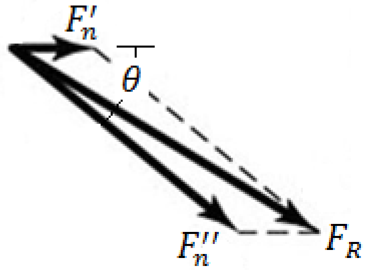
where n is the number of fasteners.



- The moment introduces secondary shear in the fasteners and the value of the secondary shear (*assuming all fatteners have the same area and same stiffness*) depends on the distance of the fastener from the center of rotation "*G*" where the closer fastener to "*G*", the less load it carries:

Secondary shear $$\boxed{F_n'' = \frac{M r_n}{r_a^2 + r_b^2 + r_c^2 + \cdots}}$$

- The two components are added using vector summation to find the magnitude of the resultant shear force on each fastener and to identify the critical fastener carrying the most amount of shear force:

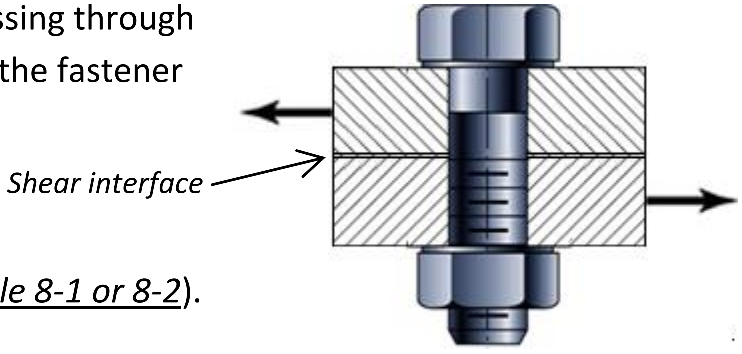


$$F_R = \left\|\vec{F}_n' + \vec{F}_n''\right\| = \sqrt{F_n'^2 + F_n''^2 + 2\,F_n'\,F_n''\,cos\,\theta}$$

- If the threaded portion of the fastener is passing through the shear interface, then the shear stress in the fastener "τ" is found as:

$$\boxed{\tau = \frac{F_R}{A_r}}$$

where A_r is the *Root Diameter Area* (Table 8-1 or 8-2).



- If the un-threaded portion of the fastener passes through the shear interface, then the shear stress is found by dividing by the nominal area A_d.

*See **Example 8-7** from text*